cfbenchmarks
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CF Bitcoin Volatility Real Time Index (BVX)

Methodology Guide

Version: 1.0

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Contents

1 Version History

Version	Date Issued	Summary of Change	Owner
v1.0	9 April 2024	N/A	CF Benchmarks Management

2 Overview

The CF Bitcoin Volatility Real Time Index (BVX) represents a single, real-time measure of implied volatility in the CME Bitcoin Options market. It is a forward-looking measure, indicating how dispersed price movements in the underlying asset may be over a given time horizon. Volatility indices in this class are often referred to as fear gauges given how they capture the stress embedded in options markets based on what market participants price into options contracts.

The BVX is constructed using tradable prices of options contracts available on CME, where price discovery is facilitated by the GLOBEX central limit order book system and transactions are centrally cleared.

The index calculation methodology is based on the pricing of variance swaps to isolate exposure to volatility of an asset, independent of the price evolution of Bitcoin prices. The weighting method of option prices based on their strikes matches the standard market approach of creating a replicating portfolio of traded options to match the payoff of a volatility swap. This allows for the creation of volatility index derivatives with constant vega over a wide span of market movements of the underlying asset.

Underlying Economic Reality

The CF Bitcoin Volatility Index is intended to measure the underlying economic reality of creating a weighted portfolio of CME Bitcoin options contracts with a view to replicating the payoff of a Bitcoin volatility swap. A volatility swap is an OTC traded financial instrument that allows an investor to gain direct linear exposure to Bitcoin volatility over a given time horizon, priced relative to a fair volatility strike defined at inception of the trade. The fair volatility strike is calculated by combining option contracts of different strike prices to produce a constant vega (volatility) exposure in the overall portfolio. This calculation is accomplished by the use of order input data from the CME that facilitate the trading of said options contracts.

3 Definitions

API: Application programming interface.

Front Contract: CME Bitcoin Futures contract which is closest to its expiry date.

Next Contract: CME Bitcoin Futures contract with expiry date after the Front Contract expiry date.

Next+1 Contract: CME Bitcoin Futures contract with expiry date immediately following the Next Contract expiry date.

Expiry Day: The last Friday of the Front Contract's month. If this is not either a UK or a U.S. business day, the contract expiry day will take place on the immediately preceding business day which is either a UK or a U.S. business day.

Expiry Datetime: 4:00 pm London on the Expiry Day.

4 Methodology

4.1 Qualitative Description

The CF Bitcoin Volatility Real Time Index is constructed using orderbook data from CME Bitcoin Futures and CME Options on Bitcoin Futures. The index is calculated using a widely accepted standard variance swap replication[1] technique, whereby option price data from different strikes and expiration dates is converted into a fair, constant maturity measure of Bitcoin volatility. The steps in the calculation of the index are as follows:

1. Collate the list of relevant CME Futures and Options contracts for the relevant calculation time and collect the corresponding orderbook data.
2. Calculate Spot Rates for every CME Futures and Options contract.
3. Select final list of option prices after filtering for erroneous data and liquidity thresholds.
4. Strip a US Dollar yield curve and calculate interest rates for a maturity equivalent to the time to expiry of the relevant options contracts.
5. Calculate the front and next term fair variance strikes, then interpolate linearly for the 30-day constant maturity Bitcoin Volatility index.

4.2 Volatility Index Calculation

The CF Bitcoin Volatility Real Time Index at Effective time is calculated as a 30-day constant maturity fair variance strike by way of linear interpolation of variance strikes calculated based on options pricing data of the two closest expirations traded on CME Globex. The variance σ_n^2 for the relevant expiration dates $n = \{1,2\}$ - where expiration date 1 immediately precedes the 30-day maturity and expiration date 2 immediately follows it - are calculated as follows:

$$\sigma_n^2 = \frac{2}{T_n} e^{r_n T_n} \sum_i \frac{\Delta K_{i,n}}{K_{i,n}^2} Q(K_{i,n}) - \frac{1}{T_n}\left(\frac{F_n}{K_{ATM,n}} - 1\right)^2$$	Eq. 1

[1] See Demeterfi et al. (1999)

Using the fair variance strikes calculated in Eq.1, the CF Bitcoin Volatility Real Time index is then determined by:

$$\sigma_{BVX,t} = 100 \times \sqrt{\left(\sigma_1^2 T_1 \frac{S_2 - S_{CM}}{S_2 - S_1} + \sigma_2^2 T_2 \frac{S_{CM} - S_1}{S_2 - S_1}\right) \times \frac{S_A}{S_{CM}}}$$ Eq. 2

where:

Symbol	*Name*	*Description*	*Type*

t	Effective time	The time as of which a CF Bitcoin Volatility Real Time index value is calculated	Parameter
S_n	Seconds to maturity	Number of seconds between the Effective time and the nth expiry date	Output
S_{CM}	Seconds to constant maturity	Number of seconds in 30 days, calculated as: $30 \times 24 \times 60 \times 60$	Input
S_A	Seconds per annum	Number of seconds in a 365 day year, calculated as: $365 \times 24 \times 60 \times 60$	Input
T_n	Years to maturity	The time to maturity of the relevant futures or options contract with the nth expiry date, calculated as: $\frac{S_n}{S_A}$	Output
r_n	Interest rate	The interest rate on the corresponding to the nth expiration date as defined by the US Dollar yield curve calculated in section 4.5.	Output
$\Delta K_{i,n}$	Strike interval	The difference between strikes relevant to strike K_i for the nth expiry date, calculated as: • Highest out-of-the-money strike K_i: $K_i - K_{i-1}$ • Lowest out-of-the-money strike K_i: $K_{i+1} - K_i$ • For all other K_i: $\frac{K_{i+1}-K_{i-1}}{2}$ For the avoidance of doubt: all relevant option contracts for the nth expiry data are ordered by strike K in ascending order, where subscript i represents the position of the individual option contract within that ordered list.	Output
$K_{ATM,n}$	ATM strike	The at-the-money strike of the options contract with the nth expiry date	Output
$\sigma_{K,n}$	Implied volatility	The implied volatility of the Call/Put option with strike K and expiry n	Output

$\boldsymbol{Q(K_n)}$	Option Spot Price	The Spot Price of the Options contract with strike price K and the nth expiry date, calculated at time t. The Spot Price $Q(K_{ATM,n})$ is calculated as the average of the Spot Prices of the ATM Put and ATM Call options.	Output
$\boldsymbol{F_n}$	Futures Spot Price	The Spot Price of the Futures contract with the nth expiry date, calculated at time t	Output

With regards to the selection of contract expiration dates to create the constant maturity index, the front contract can only be used in the calculation until up to the Front Contract Inclusion Threshold (as defined in section 6) ahead of the expiration date of the contract due to erratic price movements and illiquidity in the leadup to expiry. After that date, the front contracts are ignored, and the next and next+1 contract expiration dates are used to interpolate the 30 day index. For the avoidance of doubt, since the index maturity is set at 30 days and the year convention is 365 days, the constant time to maturity in years is calculated as 30/365 ≈ 0.082192 years.

4.3 Spot Price Determination

Spot Prices for CME Bitcoin Futures Contracts and CME Bitcoin Option Contracts are calculated as per the "CF SPOT RATES" methodology available here. The relevant set of parameters used for calculating Spot Prices of both options and futures contracts for the calculation of the CF Bitcoin Volatility Real Time index are listed in section 6 of this methodology. If no Spot Price can be determined for the Effective Time for a specific futures or options contract, either because

1. There are either no bid or no ask prices, or both,
2. The relevant orderbook is in crossed format (i.e. the bid-ask spread is not positive),

then the Spot Price for the relevant futures or options contract will be considered not viable and set to zero for index calculation purposes.

4.4 Option Strike Selection

As an initial step, all available option contracts with an expiry equal to either the expiry date of the front contract or the next contract are selected. The strikes available for trading on any given trading day can be viewed on the CME website here.

For each of the two relevant expiry dates individually, a set of options is selected based on the following steps:

1. Determine the at-the-money (ATM) strike: The ATM strike is calculated as the strike which is nearest to the Spot Price of the futures contract with matching expiry date, calculated at the same effective time as the relevant option contracts.
2. Remove all Put options with strikes greater than the ATM, and remove all Call options with strikes smaller than the ATM.
3. From this filtered list, remove any option contract with a Delta (BD) that is smaller than the Delta Threshold (as defined in section 6). Option contracts with very low Deltas are considered to be too volatile and pricing information too unreliable for inclusion in the index calculation. The Delta is calculated as follows for Call (C) and Puts (P) respectively:

$$BD_{C,K,n} = N(d1)$$

$$BD_{P,K,n} = abs(N(d1) - 1)$$

$$d1_{K,n} = \frac{ln\left(\frac{F_n}{K_n}\right) + \sigma_{K,n}^2 \frac{T_n}{2}}{\sigma_{K,n}\sqrt{T_n}}$$

Eq. 3

where N() is the cumulative standard normal distribution function and the implied volatility $\sigma_{K,n}$ is computed algorithmically by using Brent's[2] method to find the root of the following function:

[2] see Brent (1973)

$$f(\sigma_{K,n}) = BlackPrice_{\sigma,K,n} - Q(K_n)$$ $$BlackPrice_{\sigma,K,n} =$$ $$e^{-r_n T_n}\left[F_n N(d1_{K,n}) - K_n N(d2_{K,n})\right]$$, if option type is Call $$e^{-r_n T_n}\left[K_n N(-d2_{K,n}) - F_n N(-d1_{K,n})\right]$$, if option type is Put $$d2_{K,n} = d1_{K,n} - \sigma_{K,n}\sqrt{T_n}$$	Eq. 4

4. From this filtered list, remove any option contract that has two (2) consecutive option strikes on both sides of said option with zero Spot Prices. For example, if the Put option with strike price 10,000 has a viable Spot Price, but the Spot Prices of Put options with strikes 9,000, 9,500, 10,500 and 11,000 all have zero Spot Prices, then the Spot Price of the 10,000 Put option is set to zero. Therefore this option price does not influence the calculation of the BVX for the respective calculation time.

The set of options selected using above steps comprise the constituent options for the volatility index calculation.

4.5 *Interest Rate Calculation*

The risk-free interest rate is calculated based on a US Dollar yield curve interpolated from the Secured Overnight Financing Rate (SOFR) and U.S. Treasury yield curve rates. The yield curve is calculated once a day at 4pm London time based on the latest publications of both input rates (from the Fed and the Treasury website). Given that the BVX index calculation methodology laid out in 4.2 is based on a continuous time framework, we require the yield curve to be made up of continuously compounded rates. Both SOFR rates and Constant maturity Treasury (CMT) yields (i.e. bond equivalent yields (BEY)) are therefore converted to annualised percentage yields (APY), and then converted to continuously compounded rates. Finally, the rates are linearly interpolated to form the final risk-free yield curve. The continuously compounded (CC) interest rates at calculation time t are calculated as follows (for tenor T in the case of Treasury rates):

$$SOFR_{CC,t} = ln\left(\left(1 + \frac{SOFR_t}{360}\right)^{360}\right) \times \frac{365}{360}$$ $$TY_{CC,t,T} = ln\left(\left(1 + \frac{BEY_{t,T}}{2}\right)^{2}\right) \times \frac{365}{360}$$	Eq. 5

The SOFR rate and TY rates calculated using Eq.5 are used to calculate the US Dollar yield curve at Effective time t by way of linear interpolation. The day count convention used in the calculation of the US Dollar yield curve is UK Money Market ACT/365. Overnight is treated as 1 actual day.

4.6 Publication

The CF Bitcoin Volatility Real Time index (BVX) is calculated and published every second on any given Index Calculation Day, subject to Contingency Calculation Rules. The final index is rounded to two decimal places.

Index Calculation Days are defined as days on which the CME is open for CME Bitcoin Futures and CME Bitcoin Options trading and Settlement Rates for the same instruments are published.

5 Contingency Calculation Rules

The methodology applies the same contingency rules for the calculation of Spot Prices of futures and options contracts as are listed in the "CF SPOT RATES" methodology.

Beyond the above, the following rules apply:

1. If no viable Spot Price can be calculated at the Effective Time for a Futures contract, the most recent viable Spot Price will be used as current Spot Price orderbook data is deemed to be potentially erroneous. If there is no viable Spot Price within 10 seconds before the Effective Time, the CF Bitcoin Volatility will not be published for that calculation time due to Calculation Failure.

2. At least two (2) out-of-the-money strikes with viable Spot Prices on both sides of the at-the-money strike are required for the index to be calculated. If this is not the case for any

given calculation time, the current Spot Price data linked to options orderbooks will be deemed to be potentially erroneous and ignored in the index calculation. Instead, the most recently published CF Bitcoin Index Value will be republished as long as the most recently published index value has a timestamp no older than 10 seconds before the Effective time. Otherwise, the index value will not be published for the given calculation time due to Calculation Failure.

3. If the required tenors on the US Dollar yield curve cannot be determined, either because the data is unavailable through the respective APIs or there is holiday calendar mismatch between Fed/Treasury and CME, then the most recently available yield curve will be used. If there is no available yield curve for two (2) consecutive index calculation days, no index value will be published due to Calculation Failure.

6 Parameters

Effective Time (t)	Approximately every second of every Index Calculation Day
Lambda (λ)	$\frac{1}{0.3\bar{v}_t}$

	Futures Contracts	***Options Contracts***
Spacing (s)	1	1
Deviation from Mid (D)	1%	10%
Order size cap	None	None
Delta Threshold	not applicable	0.05 (also known as 5 Delta)
Front Contract Inclusion Threshold	3 calendar days (3*24*60*60 seconds)	

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